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T-Duality, Space-time Spinors and R-R Fields in Curved Backgrounds

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Abstract

We obtain the T-duality transformations of space-time spinors (the supersymmetry transformation parameters, gravitinos and dilatinos) of type-II theories in curved backgrounds with an isometry. The transformation of the spinor index is shown to be a consequence of the twist that T-duality introduces between the left and right-moving local Lorentz frames. The result is then used to derive the T-duality action on Ramond-Ramond field strengths and potentials in a simple way. We also discuss the massive IIA theory and, using duality, give a short derivation of "mass"-dependent terms in the Wess-Zumino actions on the D-brane worldvolumes.

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1 Introduction

The action of a T-duality transformation on the string worldsheet fermions can be studied by demanding compatibility with the $N = 1$ worldsheet supersymmetry. This determines the T-duality transformation of the worldsheet spinors both in flat space [1] as well as in the presence of background fields with an isometry along which duality is performed [2]. The effect on extended worldsheet supersymmetry has been studied in [3, 4], and also in [2, 5] when the extended supersymmetry does not respect the isometry. In this paper, we study the action of T-duality on space-time fermions in Type-II superstring theories with background fields and use the results to give a simple derivation of the R-R T-duality rules.

In flat backgrounds, the action of T-duality on space-time spinors follows in a rather straightforward way from its action on the worldsheet currents $\partial_\pm X^M$ and worldsheet fermions $\psi_\pm^M$ [1, 6, 7]: A T-duality with respect to X^9 sends $\partial_+ X^9 \to -\partial_+ X^9$ and $\psi_+^9 \to -\psi_+^9$, keeping all other variables unchanged. Hence, on the left-moving half of the worldsheet theory, it can be regarded as a parity reflection along X^9, while the right-moving sector remains invariant. The action of such a transformation on the left-moving Ramond ground state is represented by $\Omega_0 = \Gamma_{11}\Gamma^9$ as this operator sends Γ^9 of the left-moving sector to $\Omega_0^{-1}\Gamma^9\Omega_0 = -\Gamma^9$, consistent with the fact that Γ^9 is the zero mode of ψ_+^9 in the Ramond sector. The action of Ω_0 can now be absorbed in the space-time spinors. For example, for the parameters of space-time supersymmetry $\epsilon_\pm$ (where the subscripts "$\pm$" refer to the worldsheet sectors in which the supersymmetry acts), this leads to $\epsilon_+ \to \Omega_0\epsilon_+$, while ϵ_- remains unchanged. One can also obtain the action of T-duality on the gravitinos $\Psi_{\pm M}$ from the invariance of their vertex operators under T-duality. Ψ_{+M} contains the left-moving R ground state and hence transforms as $\Psi_{+M} \to \Omega_0\Psi_{+M}$ while Ψ_{-M} contains a left-moving NS field ψ_-^M and hence $\Psi_{-9} \to -\Psi_{-9}$, $\Psi_{-i} \to \Psi_{-i}$.

In the general case of non-flat backgrounds with an isometry, say, along X^9, T-duality no longer reduces to a parity transformation acting on left-moving (or right-moving) worldsheet variables alone. In fact, in general, it acts as a canonical transformation affecting both left and right moving sectors of the worldsheet theory [8, 9, 2]. Furthermore, in curved backgrounds, the relationship between the worldsheet fermions and space-time Dirac algebra is not as straightforward as in flat space. Therefore, it does not seem possible to obtain the T-duality action on space-time fermions, or equivalently, on the Ramond sector, from worldsheet considerations alone.

In this paper, we study the action of T-duality on space-time spinors in type-II string theories in the presence of NS-NS and R-R background fields. The spinors we consider are the space-time supersymmetry transformation parameters $\epsilon_\pm$, the two gravitinos $\Psi_{M\pm}$ and the two dilatinos $\lambda_\pm$. These results are then used to derive the T-duality rules for the R-R fields, including the massive IIA case. Both backgrounds and spinors are assumed to be independent of the coordinate X^9 along which T-duality is performed (with the exception of type-IIB potentials dual to massive type-IIA).

The paper is organized as follows: In section 2, the transformation of $\epsilon_\pm$ is obtained by identifying a T-duality action on the local Lorentz frame associated with the left-moving sector of the worldsheet theory. We also describe a set of variables in terms of which the curved-space duality resembles the flat-space case. In section 3, we consider space-time supersymmetry transformations in type-II theories in NS-NS backgrounds and determine the gravitino and dilatino T-duality transformations. These are shown to be independent of R-R backgrounds. In section 4, we use these transformations to derive the T-duality rules for R-R field strengths and potentials, emphasizing the locality of potentials in the massive type-IIA case. We then use T-duality to give a simple derivation of the "mass"-dependent terms in the Wess-Zumino action for D-branes in massive IIA theory. Section 5 contains the conclusions. Many of the formulas used in this paper are given in the appendix for convenience and to insure consistency of conventions.

2 Representation of T-duality on Spinors in Curved Backgrounds

In this section we describe how T-duality acts on the spinorial index of space-time fermions in type-II theories with background fields (the extension to other string theories is straightforward). This fully determines the transformation of the supersymmetry transformation parameters $\epsilon_\pm$.

The action of T-duality on massless NS-NS sector fields G_{MN}, B_{MN} and the dilaton ϕ is well known [12]. For later reference, we write the result here in our T-duality conventions,

$$\begin{aligned}
\widetilde{G}_{99} &= G_{99}^{-1}\,, \\
\widetilde{G}_{9i} &= -G_{99}^{-1} B_{9i}\,, \\
\widetilde{B}_{9i} &= -G_{99}^{-1} G_{9i}\,, \\
\widetilde{G}_{ij} &= G_{ij} - G_{99}^{-1}(G_{9i}G_{9j} - B_{9i}B_{9j})\,, \\
\widetilde{B}_{ij} &= B_{ij} - G_{99}^{-1}(G_{9i}B_{9j} - B_{9i}G_{9j})\,, \\
2\,\widetilde{\phi} &= 2\,\phi - \ln G_{99}\,.
\end{aligned} \tag{1}$$

Here, M, N are space-time indices in 10 dimensions. The backgrounds are assumed to be independent of the X^9 coordinate along which T-duality is performed, but may depend on the remaining coordinates which we label by X^i with $i = 0, 1, \cdots, 8$. Throughout this paper, a tilde denotes a field in the T-dual theory.

Let us decompose the 10 dimensional metric of type-II theories in terms of the vielbeins, $G_{MN} = e_M^a \eta_{ab} e^b{}_N$, where a, b are Lorentz frame indices. It is known that the T-dual theory contains two possible vielbeins that we denote by $\widetilde{e}^M_{(-)a}$ and $\widetilde{e}^M_{(+)a}$, both giving rise to the same T-dual metric $\widetilde{G}^{MN}$ [2, 4, 10]. Explicitly,

$$\widetilde{e}^M_{(-)a} = Q^M_{-N} e^N{}_a \,, \qquad \widetilde{e}^M_{(+)a} = Q^M_{+N} e^N{}_a \,. \tag{2}$$

The matrices $Q_\pm$ that implement T-duality on the vielbeins are given (along with their inverses) by,

$$Q_\pm = \begin{pmatrix} \mp G_{99} & \mp(G \mp B)_{9i} \\ \\ 0 & \mathbf{1}_9 \end{pmatrix} , \qquad Q^{-1}_\pm = \begin{pmatrix} \mp G^{-1}_{99} & -G^{-1}_{99}(G \mp B)_{9i} \\ \\ 0 & \mathbf{1}_9 \end{pmatrix} , \tag{3}$$

where $\mathbf{1}_9$ denotes the identity matrix in nine dimensions. The two vielbeins in the dual theory are related by a local Lorentz transformation $\Lambda^a_{\ b}$,

$$\widetilde{e}^M_{(+)b} = \widetilde{e}^M_{(-)a} \Lambda^a{}_b \,, \qquad \Lambda = e^{-1} Q^{-1}_- Q_+ e \,. \tag{4}$$

Using the expressions for $Q_\pm$, it is easy to see that the matrix Λ is given by

$$\Lambda^a{}_b = \delta^a{}_b - 2G^{-1}_{99} e^a{}_9 e_{9b} \,. \tag{5}$$

Note that $det\, \Lambda = -1$.

The appearance of two possible vielbeins in the dual theory is not an inconsequential ambiguity and disregarding either of them will lead to an inconsistent theory. In fact, it forces us to augment T-duality with a local Lorentz transformation acting only on the Lorentz frame associated with the left-moving sector of the worldsheet theory. To see this, it is useful to regard the two vielbeins in $G_{MN} = e_M^a \, \eta_{ab} \, e^b{}_N$ as the wavefunctions associated with the left-moving and right-moving worldsheet operators, respectively, that contribute to the graviton vertex operator. Though these vielbeins may be assigned to different worldsheet sectors, they are identical from the point of view of space-time geometry which does not directly see the string worldsheet. However, T-duality acts differently on the two worldsheet sectors and one may expect it to transform the corresponding vielbeins in different ways [1]. That this is the origin of the difference between $\widetilde{e}_{(+)}$ and $\widetilde{e}_{(-)}$ can be argued as follows: The left-moving and right-moving worldsheet sectors are interchanged under the worldsheet parity transformation $\sigma \to -\sigma$ which also interchanges Q_+ and Q_- [4] and, hence, the two vielbeins in the dual theory. This suggests that $\widetilde{e}^M_{(+)a}$ is T-dual to the vielbein associated with the left-moving sector of the original worldsheet theory, while $\widetilde{e}^M_{(-)a}$ is T-dual to the one associated with the right-moving sector. This identification also gives a heuristic understanding of the T-duality action (2) on the vielbeins: Note that

[1]That T-duality could transform the vielbeins associated with the left- and right-moving worldsheet sectors in different ways, is not evident from the transformation of the metric. This is because the T-duality action on the metric is determined by the invariance of the energy-momentum tensor and not that of the worldsheet Lagrangian.

in flat space, e^a_M's appear as wavefunctions for states created by Fourier modes of the worldsheet fermions $\psi^M_\pm$. In curved backgrounds, T-duality transforms these fermions to $\widetilde{\psi}^M_\pm = Q^M_{\pm N}\psi^N_\pm$ [2], which is consistent with the mapping of their associated wavefunctions to $\widetilde{e}_{(\pm)}$, as given by (2), depending on the worldsheet sector they come from.

The necessity of retaining both $\widetilde{e}_{(+)}$ and $\widetilde{e}_{(-)}$ in the dual theory is not evident if we are dealing with bosonic fields alone. However, their presence is essential to insure the consistency of the dual theory in the presence of space-time fermions, as will be seen in the next section[2]. This implies that we have to keep track of how the vielbeins transform, depending on the worldsheet sector they originate in. Then, to reconcile the results with the standard formulation of gravity with one set of vielbeins, we should use (4) to re-express one of the vielbeins, say $\widetilde{e}_{(+)}$ in terms of the other one, *i.e.*, $\widetilde{e}_{(-)}$. In other words, we have to augment the T-duality action on the left-moving vielbein by a local Lorentz transformation, $e \to Q_+ e\Lambda^{-1}$, so that it transforms to $\widetilde{e}_-$, rather than to $\widetilde{e}_+$. This translates to the T-duality action on the spinor index that the left-moving Ramond sector contributes to the space-time fields. Formulating the dual theory in terms of $\widetilde{e}_{(-)}$ is natural since for self-dual backgrounds, Q_- in (3) reduces to the identity matrix and $\widetilde{e}_{(-)} = e$ without further field redefinitions (Though this is not the case with $\widetilde{e}_{(+)}$, choosing it will also lead to a physically equivalent description).

Consider the space-time supersymmetry transformation parameters $\epsilon_\pm$ and the Dirac matrices $\Gamma^M = e^M_a\Gamma^a$ in either IIA or IIB theory. The Majorana-Weyl spinors $\epsilon_\pm$ are taken to be independent of X^9 and the subscripts "$\pm$" refer to their worldsheet origin and not their space-time chirality which will depend on the theory and will be specified later. After T-duality, we will have two possible sets of Γ-matrices,

$$\widetilde{\Gamma}^M_{(+)} = \widetilde{e}^M_{(+)a}\Gamma^a\,, \qquad \widetilde{\Gamma}^M_{(-)} = \widetilde{e}^M_{(-)a}\Gamma^a\,. \tag{6}$$

Keeping track of their worldsheet origin, the spinors $\epsilon_\pm$ in the dual theory are associated with the Dirac algebras generated by $\widetilde{\Gamma}^M_{(\pm)}$, respectively. The two sets of Dirac matrices are related by,

$$\widetilde{\Gamma}^M_{(+)} = \Omega^{-1}\widetilde{\Gamma}^M_{(-)}\Omega\,, \qquad \text{with}\,, \qquad \Omega^{-1}\Gamma^a\Omega = \Lambda^a{}_b\Gamma^b\,. \tag{7}$$

Clearly, Ω is the spinorial representation of the Lorentz transformation (4). The form of Ω, including its normalization, can be determined by the following argument: Let us write the $\Lambda^a{}_b$ in (5) as

$$\Lambda^a{}_b = \delta^a{}_b - 2\omega^a{}_b\,, \qquad \text{with}\,, \qquad \omega^a{}_b = G^{-1}_{99}e^a{}_9 e_{9b}\,. \tag{8}$$

One can easily verify that $\omega^a{}_b\omega^b{}_c = \omega^a{}_c$, so that $\omega^a{}_b$ is a projection operator of rank 1. The operator $\omega = \omega^a{}_b(\partial/\partial X^a)dX^b$ projects the vector $\Gamma = \Gamma^a\partial/\partial X^a$ along the isometry

[2]That both vielbeins necessarily appear in the dual theory also follows from the T-duality action on complex structures associated with extended worldsheet supersymmetry, in cases where the complex structures could be constructed in terms of target-space Killing spinors (for example, in non-compact Calabi-Yau in 4-dimensions [13]) as discussed in [2].

generator K which, normalized to unity, is given by $K = G_{99}^{-1/2} e^a{}_9 (\partial/\partial X^a)$. The projected component of Γ is then given by $< K\,, \omega\cdot\Gamma >$. The transformation $\Lambda^a_{\ b}$ in (8) changes the sign of this component, keeping other components of Γ unchanged. Therefore, its spinor representation Ω is obtained by multiplying the projected component with Γ_{11},

$$\Omega \,=\, \Gamma_{11} < K\,, \omega\cdot\Gamma >= \sqrt{G_{99}^{-1}}\,\Gamma_{11}\Gamma_9\,, \tag{9}$$

as can be directly verified using (7). The sign of Ω is not fixed by these considerations and its arbitrariness gives rise to different T-duality conventions as we will discuss later. Note the appearance of $\Gamma_9 = G_{9M}\Gamma^M$ rather than Γ^9 (as a naive generalization from the flat-space case may suggest) in this formula. This is related to the fact that, unlike in flat backgrounds (or more generally, self-dual backgrounds defined by $Q_- = 1$), T-duality now mixes $\partial_\pm X^9$ with other coordinates $\partial_\pm X^i$ when regarded as a canonical transformation in the worldsheet theory. As will be apparent in section 4, The factor $\sqrt{G_{99}^{-1}}$ in (9) is essential for giving the correct dilaton transformation, though here its existence was dictated by different considerations.

To write the dual theory with a single Dirac algebra basis, we express $\widetilde{\Gamma}^M_{(+)}$ in terms of $\widetilde{\Gamma}^M_{(-)}$ using (7), and absorb Ω in a redefinition of the spinor ϵ_+, with ϵ_- remaining unchanged. This gives the T-duality transformation rules for the space-time supersymmetry parameters $\epsilon_\pm$, which are the simplest spinorial objects in the theory, as

$$\begin{aligned} \widetilde{\epsilon}_- &= \epsilon_-\,, \\ \widetilde{\epsilon}_+ &= a_{(o-f)}\,\Omega\,\epsilon_+\,, \qquad \text{where}\,, \qquad a_{(o-f)} = \pm 1\,. \end{aligned} \tag{10}$$

Note that $\widetilde{\epsilon}_+$ and ϵ_+ have opposite space-time chiralities, which is the basis of IIA-IIB interchange under T-duality. The factor $a_{(o-f)}$ (with "o" standing for *original* and "*f*" for *final*) reflects the arbitrariness in the sign of Ω. It is used to denote $a_{(A-B)}$ when T-duality converts an original IIA theory to a final IIB theory, and $a_{(B-A)}$ *vice versa*. The arbitrariness in sign allows for two distinct conventions: Consider two successive T-duality transformations along X^9. Since $\widetilde{\Omega} = \Omega$, as can be verified using (1), we have $\widetilde{\Omega}\Omega = -1$. If we choose the convention $a_{(A-B)} = a_{(B-A)}$, then $\widetilde{\widetilde{\epsilon}}_+ = -\epsilon_+$. In fact, with this convention, all left-moving Ramond states behave in this way and T-duality squares to $(-1)^{F_L}$ on the spectrum (where F_L is the left-moving space-time fermion number). However, since IIA and IIB are different theories, we can also choose the alternative convention,

$$a_{(A-B)} = -a_{(B-A)}\,, \tag{11}$$

in which case, the T-duality operation that takes IIA to IIB is the inverse of the one that takes IIB to IIA, and the transformation squares to $+1$ on the spectrum. In the following, we use the latter convention whenever a convention is explicitly specified. The correctness of equations (10) will be checked in the next section when we examine the supersymmetry variations of gravitinos and dilatinos to extract their T-duality transformations.

Unlike the flat-space case, in non self-dual backgrounds the canonical transformation that implements T-duality acts on both worldsheet sectors. Explicitly [2],

$$\begin{aligned} \widetilde{\psi}^M_\pm &= Q^M_{\pm N}\psi^N_\pm\,, \\ \partial_\pm \widetilde{X}^M &= Q^M_{\pm N}\partial_\pm X^N + \psi^i_\pm \partial_i Q^M_{\pm N}\psi^N_\pm\,. \end{aligned} \tag{12}$$

These equations are non-trivial only for $\partial_\pm \widetilde{X}^9$ and $\widetilde{\psi}^9_\pm$, reducing to $\widetilde{X}^i = X^i$ and $\widetilde{\psi}^i_\pm = \psi^i_\pm$ for the rest. However, the invariance of ϵ_- may tempt one to search for variables in terms of which T-duality in curved space has the same form as that in flat-space, affecting only the left moving sector. To find such variables, note that the matrices $Q_\pm$, which have a very simple upper triangular form, may be decomposed as

$$Q_+ = \widetilde{A}_+^{-1}\begin{pmatrix} -1 & 0 \\ 0 & \mathbf{1}_9 \end{pmatrix} A_+\,, \qquad\qquad Q_- = \widetilde{A}_-^{-1} A_-\,.$$

Here $\widetilde{A}_\pm$ are the same matrices as $A_\pm$, but in the dual theory. These equations admit many solutions, all with $A^i_{\pm N} = \delta^i_N$ while $A^9_{\pm M}$ are not uniquely determined (for example, $A^9_{-M} = A^9_{+M} = G_{9M}/\sqrt{G_{99}}$). If we define new worldsheet fermionic and bosonic variables,

$$\begin{aligned} \Sigma^M_\pm &= A^M_{\pm N}\psi^N_\pm\,, \\ J^M_\pm &= A^M_{\pm N}\partial_\pm X^N + \psi^j_\pm\,\partial_j A^M_{\pm N}\,\psi^N_\pm\,, \end{aligned}$$

then the canonical transformations (12) implementing T-duality take the flat-background form,

$$\widetilde{J}^9_+ = -J^9_+\,, \qquad\qquad \widetilde{\Sigma}^9_+ = -\Sigma^9_+\,,$$

with J^9_- and Σ^9_- unchanged ($J^i_\pm = \partial_\pm X^i$ and $\Sigma^i_\pm = \psi^i_\pm$ are trivially invariant). However, the Lagrangian in terms of the new variables does not look any simpler which shows the basic difference between the self-dual ($Q_- = 1$), and the more general non self-dual cases, even though the transformations can be written is a similar form.

3 Action of T-duality on Gravitinos and Dilatinos

In this section we will derive the transformations of the type-II superstring gravitinos $\Psi_{\pm M}$ (not to be confused with the worldsheet spinors $\psi^M_\pm$) and dilatinos $\lambda_\pm$ under T-duality, by demanding compatibility between T-duality and space-time supersymmetry. Again, the "$\pm$" subscripts refer to the worldsheet sector in which the spinor index of the fermion, *i.e.* its Ramond component, originates and not to its space-time chirality. All spinors are assumed to be independent of the coordinate X^9 along which T-duality is performed. The T-duality action on these spinors is independent of the R-R fields, which we set to zero in this section for convenience. The case of non-zero R-R fields will be considered in the next section.

Let us first consider the supersymmetry variations of the gravitinos $\Psi_{\pm M}$. With $\epsilon_\pm$ as the supersymmetry transformation parameters and in the absence of R-R fields, these are given by

$$\begin{aligned} \delta_\pm \Psi_{\pm M} &= \left(\partial_M + \frac{1}{4} W^\pm_{Mab}\,\Gamma^{ab}\right)\epsilon_\pm \ + \cdots\,, &(13)\\ \delta_\pm \Psi_{\mp M} &= 0 \ + \cdots\,. &(14)\end{aligned}$$

Here, "$\cdots$" indicates the presence of 3-spinor terms that we do not write down explicitly, but which will be automatically accounted for in our final result. $W^\pm_{Mab}$ are the torsionful spin-connections given by

$$W^\pm_{Mab} = w_{Mab} \mp \frac{1}{2} H_{Mab}\,. \qquad (15)$$

The above transformations hold in both IIA and IIB theories, depending on the chirality of the spinors. In our conventions, in IIB, $\epsilon_\pm$ and hence $\Psi_{\pm M}$ have positive chirality while in IIA, ϵ_-, Ψ_{-M} have positive chirality and ϵ_+, Ψ_{+M} have negative chirality. The supersymmetry transformation generated by ϵ_+ (ϵ_-) acts on the left-moving (right-moving) worldsheet sector by interchanging R and NS boundary conditions. Therefore, the supersymmetry variations $\delta_\pm\Psi_{\pm M}$ convert R-NS states into NS-NS states and do not get modified if R-R fields are switched on. Therefore, we expect that the gravitino T-duality rules obtained from equation (13) are independent of R-R fields. The same argument applies to dilatino T-duality rules.

Let us now consider the gravitino supersymmetry variations in the T-dual theory. First, note that the dual theory contains two sets of torsionful spin-connections, corresponding to the two vielbeins $\widetilde{e}^a_{(-)M}$ and $\widetilde{e}^a_{(+)M}$ given by (2). We denote these by $\widetilde{W}^\pm_{(-)Mab}$ and $\widetilde{W}^\pm_{(+)Mab}$, respectively. One can verify that

$$\begin{aligned} \widetilde{W}^-_{(-)Mab} &= W^-_{Nab}(Q_+^{-1})^N{}_M\,, &(16)\\ \widetilde{W}^+_{(+)Mab} &= W^+_{Nab}(Q_-^{-1})^N{}_M\,. &(17)\end{aligned}$$

Since we have chosen to express the T-dual theory in terms of $\widetilde{e}_{(-)}$, the supersymmetry variations $\delta_\pm\widetilde{\Psi}_{\pm M}$ in the T-dual theory should be expressed in terms of $\widetilde{W}^\pm_{(-)Mab}$ alone,

$$\delta_\pm\widetilde{\Psi}_{\pm M} = \left(\partial_M + \frac{1}{4}\widetilde{W}^\pm_{(-)Mab}\Gamma^{ab}\right)\widetilde{\epsilon}_\pm \ + \cdots\,. \qquad (18)$$

To determine $\widetilde{\Psi}_{+M}$ in terms of Ψ_{+M}, note that using the relation (4) between $\widetilde{e}_{(+)}$ and $\widetilde{e}_{(-)}$, we can write $\widetilde{W}^+_{(+)Mab}$ in terms of $\widetilde{W}^+_{(-)Mab}$ as,

$$\begin{aligned} \widetilde{W}^+_{(+)M}{}^a{}_b\Gamma_a{}^b &= \widetilde{W}^+_{(-)M}{}^c{}_d\,(\Lambda^{-1})^a{}_c\,\Lambda^d{}_b\,\Gamma_a{}^b + (\Lambda^{-1})^a{}_c\partial_M\Lambda^c{}_b\,\Gamma_a{}^b \\ &= \widetilde{W}^+_{(-)M}{}^a{}_b\Omega^{-1}\Gamma_a{}^b\Omega + 4\Omega^{-1}\partial_M\Omega\,. &(19)\end{aligned}$$

Now, using equations (10) and (16-19), along with the fact that $Q^{i}_{\pm j} = \delta^{i}{}_{j}$ (3) and $\partial_9 \epsilon_- = 0$, it is easy to see that the variation (13) implies the one in the dual theory (18) provided,

$$\begin{aligned} \delta_- \widetilde{\Psi}_{-M} &= \delta_- \Psi_{-N} \, (Q_+^{-1})^N{}_M \, + \cdots \,, & (20) \\ \delta_+ \widetilde{\Psi}_{+M} &= a_{(o-f)} \, \Omega \, \delta_+ \Psi_{+N} \, (Q_-^{-1})^N{}_M \, + \cdots \,. & (21) \end{aligned}$$

Again, "$\cdots$" denotes 3-spinor terms.

Let us now consider the supersymmetry variations of the dilatinos $\lambda_\pm$ in the absence of R-R fields,

$$\begin{aligned} \delta_\pm \lambda_\pm &= \frac{1}{2} \left(\Gamma^M \partial_M \phi \mp \frac{1}{12} \Gamma^{MNK} H_{MNK} \right) \epsilon_\pm \, + \cdots \,, & (22) \\ \delta_\pm \lambda_\mp &= 0 \, + \cdots \,. & (23) \end{aligned}$$

These are again valid in both IIA and IIB theories. In IIB, both dilatinos have negative chirality, while in IIA, λ_- has negative chirality and λ_+ has positive chirality. Switching on R-R fields does not affect equation (22). In the T-dual theory, written in terms of the vielbein $e^a_{(-)M}$, the variations $\delta_\pm \widetilde{\lambda}_\pm$ are given by

$$\delta_\pm \widetilde{\lambda}_\pm = \frac{1}{2} \left(\widetilde{\Gamma}^M_{(-)} \partial_M \widetilde{\phi} \mp \frac{1}{12} \widetilde{\Gamma}^{MNK}_{(-)} \widetilde{H}_{MNK} \right) \widetilde{\epsilon}_\pm \, + \cdots \,. \tag{24}$$

Using $\widetilde{\phi} = \phi - \frac{1}{2} \ln G_{99}$ and

$$\widetilde{\Gamma}^{MNK}_{(\mp)} \widetilde{H}_{MNK} = \Gamma^{MNK} H_{MNK} \mp 6 \, G_{99}^{-1} \, \Gamma_9 \, (W^{\mp}_{9ab} \, \Gamma^{ab}) \pm 6 \, G_{99}^{-1} \, \Gamma^i \, \partial_i \, G_{99} \,, \tag{25}$$

along with equation (10), one can see that the supersymmetry variations (22) and (24) are compatible provided

$$\begin{aligned} \delta_- \widetilde{\lambda}_- &= \delta_- \lambda_- - G_{99}^{-1} \, \Gamma_9 \, \delta_- \Psi_{-9} \, + \cdots \,, & (26) \\ \delta_+ \widetilde{\lambda}_+ &= a_{(o-f)} \, \Omega \, \left(\delta_+ \lambda_+ - G_{99}^{-1} \, \Gamma_9 \, \delta_+ \Psi_{+9} \right) \, + \cdots \,. & (27) \end{aligned}$$

Equations (20),(21) and (26),(27) give the T-duality transformations of the supersymmetry variations $\delta \Psi_{\pm M}$ and $\delta \lambda_\pm$ to linear order in spinors and receive corrections cubic in the spinors whose presence is indicated by "$\cdots$". From these we can read off the T-duality transformations of the gravitinos and dilatinos, in principle, only to linear order in the spinors. However, as we will show, the linear order result is exact and in fact, it dictates the form of the 3-spinor corrections to the T-duality maps for the supersymmetry variations above. Thus, for the gravitinos $\Psi_{\pm M}$, we have the T-duality transformations

$$\begin{aligned} \widetilde{\Psi}_{-M} &= \Psi_{-N} \, (Q_+^{-1})^N{}_M \,, \\ \widetilde{\Psi}_{+M} &= a_{(o-f)} \, \Omega \, \Psi_{+N} \, (Q_-^{-1})^N{}_M \,, \end{aligned} \tag{28}$$

and for the dilatinos $\lambda_\pm$ we have the transformations

$$\begin{array}{rcl}\widetilde{\lambda}_- & = & \lambda_- - G_{99}^{-1}\,\Gamma_9\,\Psi_{-9}\,, \\ \widetilde{\lambda}_+ & = & a_{(o-f)}\,\Omega\left(\lambda_+ - G_{99}^{-1}\,\Gamma_9\,\Psi_{+9}\right)\,.\end{array} \tag{29}$$

Here, Ω is given by (9) and, as described below equation (10), $a_{(o-f)} = \pm 1$ stands for $a_{(A-B)}$ if T-duality takes us from IIA to IIB, and for $a_{(B-A)}$ if it acts the other way round. Setting $a_{(A-B)} = -a_{(B-A)}$ insures that T-duality squares to $+1$ on the spectrum.

That equations (28) and (29) do not receive corrections can be seen as follows: To linear order in spinors, these equations are uniquely determined by (20), (21) and (26), (27) thus only leaving the possibility of adding corrections cubic in the spinors. The presence of such terms, however, can be ruled out on general grounds as they would give rise to derivative interactions for the spinors in the dual supergravity action. To rule out, in a more concrete way, the existence of both 3-spinor corrections, as well as corrections proportional to R-R fields, we consider the supersymmetry variations of the NS-NS fields G_{MN}, B_{MN} and ϕ given by equations (C.6) in the appendix. These variations contain no R-R fields and are only bilinear in spinors. Using (1) along with (28) and (29) one can easily verify that that these variations are consistent with T-duality. On the other hand, if (28) and (29) contained either 3-spinor terms or R-R dependent terms, this would not be the case. This establishes that the spinor T-duality rules given above are exact. Note that the NS-NS supersymmetry variations (C.6) are insensitive to the multiplicative factor Ω. Therefore, while they can be used to rule out additional additive contributions to (28) and (29), they cannot be used to infer the existence of Ω in these transformations.

For supersymmetric backgrounds, when the fermionic backgrounds $\Psi_\pm$ and $\lambda_\pm$ along with their supersymmetry variations are set to zero, equations (13) and (22) reduce to the string theoretic Killing spinor equations for $\epsilon_\pm$. Equations (28) and (29) are then trivial for the background spinors, but can be used to obtain the T-duality transformation of the fermionic excitations around supersymmetric backgrounds. In some cases, when the Killing spinor itself does not transform (as is the case with ϵ_-), the compatibility of the Killing spinor equation with T-duality was investigated in [10, 11].

4 R-R T-duality Revisited

As shown above, the T-duality rules for space-time fermions do not depend on the R-R fields. In this section we use these rules, along with the requirement of compatibility of T-duality with space-time supersymmetry, to determine the T-duality rules for R-R fields and discuss some related issues. Most of the results in this section are not new but are re-derived here in a unified and more convenient way. The IIA/IIB T-duality rules for R-R fields were derived in [14, 15] by studying the supergravity action and equations of motion (also see [16]) and in [17, 18, 19] by dimensional reduction of the Wess-Zumino term in the D-brane worldvolume action, both considering the bosonic sector alone. Our derivation of these rules here emphasizes the compatibility of the T-duality conventions used for the R-R fields with those used for the spinors. The T-duality rules relating IIB to the massive IIA theory were obtained in [15, 17, 18]. Here we re-derive these rules for generic configurations, emphasizing how potential non-localities in the T-duality rules for R-R potentials are avoided. We also present a simple derivation of the "mass"-dependent terms in the Wess-Zumino action for "massive" IIA branes using T-duality.

In the presence of R-R backgrounds, the supersymmetry variations $\delta_+\Psi_{+M}$ and $\delta_-\Psi_{-M}$ are still given by (13), while $\delta_-\Psi_{+M}$ and $\delta_+\Psi_{-M}$ are no longer zero and receive contributions from R-R fields. The same is true for the dilatino variations $\delta_\pm\lambda_\mp$. The T-duality rules for the R-R fields can be obtained by considering any one of these variations, say $\delta_-\Psi_{+M}$. In type IIA theory, this variation is given by [20] (see appendix C for details),

$$\delta_-\Psi_{+M} = \frac{1}{8}e^{\phi}\Big[F^{(0)} + \frac{1}{2!}\Gamma^{M_1M_2}F^{(2)}_{M_1M_2} + \frac{1}{4!}\Gamma^{M_1M_2M_3M_4}F^{(4)}_{M_1M_2M_3M_4}\Big]\Gamma_M\epsilon_- + \cdots\,, \tag{30}$$

where "$\cdots$" denote 3-spinor terms as usual. $F^{(0)} = m$ is the mass parameter of massive type-IIA theory and the field strengths $F^{(n)}$ for the massive theory are given by (C.3) in the appendix. The usual massless IIA equations are obtained by setting $m = 0$. In type-IIB theory the corresponding variation is given by [21] (see appendix B for details),

$$\begin{aligned}\delta_-\Psi_{+M} = -\frac{1}{8}e^{\phi}\Big[&\Gamma^{M_1}F^{(1)}_{M_1} + \frac{1}{3!}\Gamma^{M_1M_2M_3}F^{(3)}_{M_1M_2M_3}\\ &+ \frac{1}{2(5!)}\Gamma^{M_1M_2M_3M_4M_5}F^{(5)}_{M_1M_2M_3M_4M_5}\Big]\Gamma_M\epsilon_- + \cdots\,.\end{aligned} \tag{31}$$

It is convenient to write these two equations in the generic form

$$\delta_-\Psi_{+M} = \frac{1}{2(8)}e^{\phi}\left[\sum_n \frac{(-1)^n}{n!}\Gamma^{M_1\cdots M_n}F^{(n)}_{M_1\cdots M_n}\right]\Gamma_M\epsilon_- + \cdots\,. \tag{32}$$

In exactly the same way as for the R-R vertex operator in flat space (see, for example, [6, 7]), the actual content of the above equation is determined by the chirality of the space-time spinors: In type-IIB theory, both ϵ_- and Ψ_{+M} have positive chirality and therefore the right hand side contains only terms with even number of Γ-matrices (corresponding to $n = 1, 3, 5, 7, 9$), whereas in IIA, ϵ_- and Ψ_{+M} have positive and negative chiralities respectively and hence only terms with even n ($n = 0, 2, 4, 6, 8, 10$) enter the summation. Furthermore, using the Γ-matrix identity (A.2), the positive chirality of ϵ_- implies that $F^{(n)} = -(-1)^{n(n-1)/2}\,{}^*F^{(10-n)}$. This allows us to write the summation in terms of $F^{(n)}$ with $n \leq 5$ alone, recovering (30) and (31).

Let us now consider the above equation in the T-dual theory expressed in terms of the vielbein $\widetilde{e}^{a}_{(-)M}$,

$$\delta_-\widetilde{\Psi}_{+M} = \frac{1}{2(8)}e^{\widetilde{\phi}}\left[\sum_n \frac{(-1)^n}{n!}\widetilde{\Gamma}^{M_1\cdots M_n}_{(-)}\widetilde{F}^{(n)}_{M_1\cdots M_n}\right]\widetilde{\Gamma}_{(-)M}\widetilde{\epsilon}_- + \cdots\,. \tag{33}$$

Using equations (2), (6), (10) and (28), one can readily obtain the T-duality transformation for the R-R field strengths as

$$\widetilde{F}^{(n)}_{M_1\cdots M_n} = (-1)^n a_{(o-f)} \left(F^{(n+1)}_{9N_1\cdots N_n} + nG_{9[N_1} F^{(n-1)}_{N_2\cdots N_n]} \right) (Q_-^{-1})^{N_1}{}_{M_1} \cdots (Q_-^{-1})^{N_n}{}_{M_n} \,. \tag{34}$$

where, $a_{(o-f)}$ denotes a convention dependent sign as explained below equation (10). Let us now choose the convention (11) so that T-duality squares to 1 on R-R fields. Then, using the form of Q_-^{-1} given in (3), the above equation reduces to the component form,

$$\widetilde{F}^{(n)}_{9i_2\cdots i_n} = -a_{(A-B)} \left[F^{(n-1)}_{i_2\cdots i_n} - (n-1) G^{-1}_{99} G_{9[i_2} F^{(n-1)}_{9i_3\cdots i_n]} \right] , \tag{35}$$

$$\widetilde{F}^{(n)}_{i_1 i_2\cdots i_n} = -a_{(A-B)} F^{(n+1)}_{9i_1\cdots i_n} - nB_{9[i_1} \widetilde{F}^{(n)}_{9i_2\cdots i_n]} \,. \tag{36}$$

$a_{(A-B)}$ is still arbitrary and could be chosen as either $+1$ or -1. The antisymmetrization denoted by the square bracket affects the indices i_n and not the index 9. Since the spinors were assumed to be independent of X^9, equation (32) implies that $F^{(n)}$ should also be independent of this coordinate.

The above T-duality rules for $F^{(n)}$ are valid for both massless and massive type-IIA theories and can be iteratively integrated to yield the corresponding transformations for the R-R potentials $C^{(n)}$. Let us first consider duality between IIB and massless IIA. In this case, $F^{(0)} = m = 0$ and the field strengths are given by (B.10). $C^{(n)}$ can be chosen to be X^9-independent and under T-duality transform as [14, 16, 15, 18, 19]

$$\widetilde{C}^{(n)}_{9i_2\cdots i_n} = a_{(A-B)} \left[C^{(n-1)}_{i_2\cdots i_n} - (n-1) G^{-1}_{99} G_{9[i_2} C^{(n-1)}_{9i_3\cdots i_n]} \right] , \tag{37}$$

$$\widetilde{C}^{(n)}_{i_1 i_2\cdots i_n} = a_{(A-B)} C^{(n+1)}_{9i_1\cdots i_n} - nB_{9[i_1} \widetilde{C}^{(n)}_{9i_2\cdots i_n]} \,. \tag{38}$$

Let us now consider the massive-IIA case. For $n = 0$, equation (36) reduces to $\widetilde{F}^{(0)} = -a_{(A-B)} F^{(1)} = -a_{(A-B)} \partial_9 C^{(0)}$. As noticed in [15, 17, 18], this implies that type-IIB theory dualizes to the massive IIA theory with $\widetilde{F}^{(0)} = m$, provided the IIB 0-form has an X^9 dependence given by $C^{(0)} = -a_{(A-B)} m X^9 + \hat{C}^{(0)}$, where the last term is independent of X^9. Naively, one may expect that this X^9-dependence could lead to a similar dependence for the IIA potentials, which should not be the case: Consider an X^9-dependent function $C(X^9)$, say, in the IIB theory leading to an X^9-dependent T-dual $\widetilde{C}(X^9)$ in IIA. Since the natural variable in the T-dual theory is $\widetilde{X}^9$, which is related to X^9 through the canonical transformation (12), $\widetilde{C}$ has to be expressed in terms of $\widetilde{X}^9$. However, the relationship between X^9 and $\widetilde{X}^9$ is non-local, involving an integration over the string worldsheet, and hence $\widetilde{C}$ is a non-local function of $\widetilde{X}^9$. This problem can be avoided if we arrange things such that the X^9-dependent C dualizes to an X^9-independent $\widetilde{C}$, or *vice versa.* Let us define

$$\widehat{C}^{(0)} = C^{(0)} + a_{(A-B)} m X^9 \,,$$
$$\widehat{C}^{(2)}_{M_1 M_2} = C^{(2)}_{M_1 M_2} + a_{(A-B)} m X^9 B_{M_1 M_2} \,,$$
$$\widehat{C}^{(4)}_{M_1\cdots M_4} = C^{(4)}_{M_1\cdots M_4} + 3a_{(A-B)} m X^9 B_{[M_1 M_2} B_{M_3 M_4]} \,,$$

or more generally, using the notation of [18], with $C = \sum_{n=0}^{9} C^{(n)}$,

$$\widehat{C} = C + a_{(A-B)}\, m\, X^9 e^B \,. \tag{39}$$

We give the $C^{(2p)}$ in type-IIB a dependence on X^9 in such a way that $\widehat{C}^{(2p)}$ are X^9 independent, while in type-IIA, $\widehat{C}^{(2p+1)} = C^{(2p+1)}$ and are X^9 indepentent. Then, using the T-duality rules for the field strengths (35,36), along with equations (B.10) for type-IIB and (C.3) for the massive type-IIA, one can obtain the T-duality rules for the potentials. These are still given by (37) and (38) but now with all $C^{(2p)}$ replaced by $\widehat{C}^{(2p)}$. The X^9 independence of $\widehat{C}^{(2p)}$ guarantees the X^9 independence of the IIA potentials $C^{(2p+1)}$, preventing the appearance of non-localities. Note that while the massive T-duality rules, written in terms of $\widehat{C}^{(2p)}$, have the same form as the usual massless IIA/IIB rules, the two differ by m-dependent terms when written in terms of the actual R-R potentials $C^{(2p)}$.

The special X^9 dependence of $C^{(2p)}$ can be easily understood when massive-IIA/IIB duality is regarded as a Scherk-Schwarz compactification to 9 dimensions [15]: The $U(1) \subset SL(2,R)$ transformation in IIB theory that gives the right X^9 dependence to $C^{(0)}$, by shifting it to $C^{(0)} - a_{(A-B)} m X^9$ (corresponding to $p = s = 1$, $r = 0$ and $q = -a_{(A-B)} m X^9$ in (B.5)), also produces the correct X^9 dependences in $C^{(2)}$ and $C^{(4)}$.

For $m \neq 0$, the Wess-Zumino terms in the IIA D-brane worldvolume actions contain m-dependent terms the forms of which were studied in [17, 18]. We will now derive these terms in a very straightforward way using T-duality: Let us start with the WZ terms in the D-brane worldvolume actions in type-IIB theory and express the potentials $C^{(2p)}$ in terms of $\widehat{C}^{(2p)}$ as defined in (39),

$$I^{(IIB)}_{WZ} = \int_{\omega_{2p}} C e^{F-B} = \int_{\omega_{2p}} \widehat{C} e^{F-B} - a_{(A-B)} m \int_{\omega_{2p}} X^9 e^F \,. \tag{40}$$

When $C^{(2p)}$ are chosen such that $\widehat{C}^{(2p)}$ are X^9 independent, the dual theory is massive type-IIA. Therefore, on dimensional reduction, $I^{(IIB)}_{WZ}$ should reduce to the corresponding action for massive IIA theory, including the m-dependent terms. As mentioned earlier, the massive T-duality rules relating $\widehat{C}^{(2p)}$ and $C^{(2p+1)}$ have exactly the same form as the massless T-duality rules relating $C^{(2p)}$ and $C^{(2p+1)}$. Therefore, the analysis for the massless case, for example, as presented in [19] or [18], implies that the first term on the right hand side of (40) dualizes to the standard WZ term in type-IIA which is common between the massive and massless theories. The m-dependent terms are contained in the second integral on the right hand side of (40). Let us identify X^9 with a worldvolume direction, say σ, along which the theory is reduced. Taking F to be Abelian ($F = \mathrm{d}V$), we write $e^F = \sum_p \frac{1}{p!} \mathrm{d}(V \wedge F^{p-1})$ so that,

$$\int_{\omega_{2p}} X^9 e^F = \sum_p \frac{1}{p!} \int_{\omega_{2p}} X^9 \left[\partial_\sigma (V \wedge F^{p-1}) \wedge \mathrm{d}\sigma + \partial_\alpha (V \wedge F^{p-1}) \wedge \mathrm{d}x^\alpha \right] , \tag{41}$$

where, x^α are the worldvolume directions transverse to σ. Since X^9 does not depend on x^α, the second term in the integrand leads to a surface term and can be dropped. The

$(2p-1)$-form $V \wedge F^{p-1}$ in the first term now only has non-zero components along x^α, and not along σ. Finally, remembering that $X^9 = \sigma$ and dropping a surface term, the integration over σ leads to

$$I_{WZ}^{(IIA)} = \int_{\omega_{2p-1}} Ce^{F-B} + a_{(A-B)} m \sum_p \frac{1}{p!} \int_{\omega_{2p-1}} V \wedge (\mathrm{d}V)^{p-1} \,, \tag{42}$$

which reproduces the m-dependent terms of [17, 18] (we have ignored the D-brane tension that can be easily inserted into the equations).

5 Conclusions

We have shown that, besides acting on the space-time indices of fields, T-duality also has an action on the local Lorentz frame associated with the left-moving sector of the world-sheet theory by twisting it with respect to the one associated with the right-moving sector. This twist translates to the T-duality action on the spinor index originating in the left-moving Ramond sector, and fixes the T-duality action on the space-time supersymmetry parameters. The gravitinos and dilatinos also contain an NS sector contribution to their T-duality transformations which is obtained by demanding consistency between T-duality and space-time supersymmetry. It is also shown that the T-duality action on the spinors is independent of the R-R backgrounds. The result is then used to re-derive the R-R T-duality rules. We discuss the case of the massive IIA theory in more detail, showing that there exist variables in terms of which the massive T-duality rules for the R-R potentials have the same form as the massless ones, manifestly avoiding non-local relations between potentials. Using this, we give a simple derivation of the "mass"-dependent terms in the WZ actions for the associated D-branes based on T-duality. In most part, we have explicitly retained the convention dependence of the T-duality action on the Ramond sector. In one convention, T-duality squares to 1, while in the other, it squares to $(-1)^{F_L}$ on the spectrum, where F_L is the left-moving space-time fermion number.

There are certain similarities between T-duality in flat and curved backgrounds. At the worldsheet level, as we have shown, there exist variables in terms of which the canonical transformation that implements T-duality in curved space, has the flat-space form. One can also check that the T-duality rules for gravitinos and R-R fields in curved backgrounds easily follow from their flat-space vertex operators, provided we interpret these operators as curved space objects (which, of course, is not really the case). For example, consider the gravitino emission operator $\sim \bar{S}_{+s} \Psi^s_{+M} \psi^M_-$ in flat space. To interpret this as a curved-space expression, we define the spin-filed S_{+s} as an operator that generates space-time supersymmetry transformations of Ψ^s_{+M} with parameter ϵ^s_+, but now in curved-space. S_+ and ϵ_+ will have opposite space-time chiralities and $\bar{S}_+ \epsilon_+$ is invariant under T-duality. Then using the curved-space T-duality rules for ϵ_+ (10) and ψ^M_- (12) in the flat-space vertex operator, we recover the T-duality action (28) on Ψ_{+M}. Similarly, the R-R T-duality rules can be obtained from the corresponding flat-space vertex operator, $e^{\phi}\bar{S}_{+s}F^{ss'}S_{-s'}$, where $F^{ss'}$ is the R-R bi-spinor.

Note Added: In a recent paper [22], which appeared after this paper was completed, the authors consider the $SO(d,d,Z)$ action on R-R fields from the point of view of low-energy effective action. One should be able to obtain the same results in our approach, after determining the $SO(d,d,Z)$ action on gravitinos, and then using space-time supersymmetry. The results are expected to look the same as the single T-duality case with $Q_{\pm}$ and Ω appropriately generalized to $SO(d,d)$.

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Appendix

A Γ-Matrix Conventions

We use the metric signature $\{-1,+1,\ldots,+1\}$ and Gamma matrix conventions

$$\{\Gamma^a,\Gamma^b\} = 2\eta^{ab}\,, \tag{A.1}$$

so that in the Majorana-Weyl representation all Γ^a are real, with Γ^0 antisymmetric and others symmetric. We also need the identity (with $\epsilon^{01\cdots 9}=1$)

$$\Gamma^{M_1\cdots M_n}\Gamma_{11} = \frac{-(-1)^{n(n-1)/2}}{\sqrt{-G}(10-n)!}\epsilon^{M_1\cdots M_{10}}\Gamma_{M_{n+1}\cdots M_{10}}\,. \tag{A.2}$$

B Type-IIB Supergravity

For the gravitino and dilatino supersymmetry variations in type-IIB supergravity, we start with the $SU(1,1)$ invariant formulation of the theory in [21]. Using a prime to indicate the use of the Einstein metric and after scaling the 5-from field strength to match the standard string theory conventions, we have

$$\delta\lambda' = i\Gamma'^{M} P_M \epsilon'^{*} - \frac{i}{24}\Gamma'^{KLN} G_{KLN}\epsilon' + \cdots\,, \tag{B.1}$$

$$\begin{aligned}\delta\Psi'_M = D_M\epsilon' &+ \frac{1}{96}\left(\Gamma'^{\;KLN}_{M} G_{KLN} - 9\Gamma'^{LN} G_{MLN}\right)\epsilon'^{*} \\ &+ \frac{i}{4(480)}\Gamma'^{KLNPQ}\Gamma'_M F_{KLNPQ}\epsilon' + \cdots\,.\end{aligned} \tag{B.2}$$

Here, ϵ', Ψ'_M and λ' are complex Weyl spinors with $\Gamma_{11}\epsilon' = \epsilon'$, $\Gamma_{11}\Psi' = \Psi'$, while λ' has negative chirality, and

$$\begin{array}{rclrcl} D_M\epsilon' &=& \left(\partial_M + \frac{1}{4}w'_{Mab}\Gamma^{ab} - \frac{i}{2}Q_M\right)\epsilon'\,, & G_{KLN} &=& -\epsilon_{\alpha\beta}V^{\alpha}_{+}F^{\beta}_{KLN}\,, \\ P_M &=& -\epsilon_{\alpha\beta}V^{\alpha}_{+}\partial_M V^{\beta}_{+}\,, & Q_M &=& -i\epsilon_{\alpha\beta}V^{\alpha}_{-}\partial_M V^{\beta}_{+}\,. \end{array} \tag{B.3}$$

$\alpha,\beta = 1,2$ are $SU(1,1)$ indices, $V^{\alpha}_{\pm}$ is an $SU(1,1)$ matrix ($V^1_-V^2_+ - V^1_+V^2_- = 1$) and $F^1_{KLN} = F^{2*}_{KLN}$. To identify the fields in the usual string theory conventions, we go to the $SL(2,R)$ formulation by writing F^{α} in the real basis. Then the NS-NS and R-R 2-forms B_{MN} and $C^{(2)}_{MN}$ are given by

$$\begin{pmatrix} -dC^{(2)} \\ dB \end{pmatrix} = \begin{pmatrix} Re(F^1) \\ Im(F^1) \end{pmatrix} = h\begin{pmatrix} F^1/\sqrt{2} \\ F^2/\sqrt{2} \end{pmatrix}\,, \quad \text{with}\,, \quad h = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & +i \end{pmatrix}\,.$$

The dilaton and the R-R scalar C^0 are identified by parameterizing the matrix V such that

$$U = hV \equiv h\begin{pmatrix} V^1_- & V^1_+ \\ V^2_- & V^2_+ \end{pmatrix} = \frac{1}{\sqrt{2\tau_2}}\begin{pmatrix} -\bar{\tau}e^{i\theta} & -\tau e^{-i\theta} \\ e^{i\theta} & e^{-i\theta} \end{pmatrix}\,, \tag{B.4}$$

with $\tau = C^{(0)} + ie^{-\phi}$. We can set $\theta = 0$ by fixing the $U(1)$. In these conventions, the $SL(2,R)$ action takes the form

$$\tau \to \frac{p\tau + q}{r\tau + s}\,, \qquad \begin{pmatrix} C^{(2)} \\ B \end{pmatrix} \to \begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} C^{(2)} \\ B \end{pmatrix}\,. \tag{B.5}$$

$C^{(4)}$ also transforms such that $F^{(5)}$ is invariant (see (B.10)). Having identified the dilaton, we define the string frame metric and associated spinors as,

$$G_{MN} = e^{\phi/2}G'_{MN}\,, \quad \epsilon = e^{\phi/8}\epsilon'\,, \quad \lambda = e^{-\phi/8}\lambda'\,, \quad \Psi_M = e^{\phi/8}(\Psi'_M + \frac{i}{4}\Gamma'_M\lambda'^{*})\,.$$

Furthermore, we write the complex Weyl spinors in terms of real Majorana-Weyl spinors, $\epsilon = \epsilon_+ + i\epsilon_-$, $\Psi_M = \Psi_{+M} + i\Psi_{-M}$, $\lambda = \lambda_- + i\lambda_+$, where the subscript "$\pm$" is chosen to denote the worldsheet sector that contributes the spin content of the spinor. The supersymmetry variations (B.2) then take the form (to linear order in the spinors),

$$\delta_\pm\lambda_\pm = \frac{1}{2}\left(\Gamma^M\partial_M\phi \mp \frac{1}{12}\Gamma^{M_1M_2M_3}H_{M_1M_2M_3}\right)\epsilon_\pm + \cdots\,, \tag{B.6}$$

$$\delta_\mp\lambda_\pm = \frac{1}{2}e^\phi\left(\pm\Gamma^M F^{(1)}_M + \frac{1}{12}\Gamma^{M_1M_2M_3}F^{(3)}_{M_1M_2M_3}\right)\epsilon_\mp + \cdots\,, \tag{B.7}$$

$$\delta_\pm\Psi_{\pm M} = \left(\partial_M + \frac{1}{4}\left(w_{Mab} \mp \frac{1}{2}H_{Mab}\right)\Gamma^{ab}\right)\epsilon_\pm \; + \; \cdots\,, \tag{B.8}$$

$$\delta_\mp\Psi_{\pm M} = \frac{1}{8}e^\phi\Big(\mp\Gamma^{M_1}F^{(1)}_{M_1} - \frac{1}{3!}\Gamma^{M_1M_2M_3}F^{(3)}_{M_1M_2M_3} \mp \frac{1}{2(5!)}\Gamma^{M_1\cdots M_5}F^{(5)}_{M_1\cdots M_5}\Big)\Gamma_M\epsilon_\mp + \cdots\,. \tag{B.9}$$

The R-R potentials $C^{(n)}$ are defined such that,

$$F^{(n)}_{M_1\cdots M_n} = n\partial_{[M_1}C^{(n-1)}_{M_2\cdots M_n]} - \frac{n!}{3!(n-3)!}H_{[M_1M_2M_3}C^{(n-3)}_{M_4\cdots M_n]}\,. \tag{B.10}$$

C Type-IIA Supergravity

The gravitino and dilatino supersymmetry variations in type-IIA theory are given in [20] for massive IIA. When written in terms of appropriate variables, they lead to the usual massless IIA equations when the mass parameter is set to zero. In the standard string theory normalizations for the fields, these equations take the form,

$$\begin{aligned}\delta\lambda' =& \frac{1}{2}\Big[\Gamma'^M\partial_M\phi - \frac{1}{12}\Gamma'^{MNP}H_{MNP}\Gamma_{11}\Big]\epsilon' \\ &+\frac{1}{8}\Big[5e^{5\phi/4}F^{(0)} - \frac{3}{2!}e^{3\phi/4}\Gamma'^{MN}F^{(2)}_{MN}\Gamma_{11} + \frac{1}{4!}e^{\phi/4}\Gamma'^{MNPQ}F^{(4)}_{MNPQ}\Big]\epsilon' + \cdots\,,\end{aligned} \tag{C.1}$$

$$\begin{aligned}\delta\Psi'_M =& \Big[\partial_M + \frac{1}{4}w'_{Mab}\Gamma^{ab} + \frac{1}{96}e^{-\phi/2}\left(\Gamma'^{\;NPQ}_M - 9\delta^{[N}_M\Gamma'^{PQ]}\right)H_{NPQ}\Gamma_{11}\Big]\epsilon' \\ &+\frac{1}{32}\Big[-\frac{1}{2}e^{5\phi/4}\Gamma'_MF^{(0)} - \frac{1}{2}e^{3\phi/4}\left(\Gamma'^{\;NP}_M - 14\delta^{[N}_M\Gamma'^{P]}\right)F^{(2)}_{NP}\Gamma_{11} \\ &+\frac{1}{4}e^{\phi/4}\left(\Gamma'^{\;NPQR}_M - \frac{20}{3}\delta^{[N}_M\Gamma'^{PQR]}\right)F^{(4)}_{NPQR}\Big]\epsilon' + \cdots\,.\end{aligned} \tag{C.2}$$

Here, a prime indicates the use of the Einstein metric, "$\cdots$" denote 3-spinor terms and the field strengths $F^{(n)}$ are given by

$$\begin{aligned}F^{(0)} &= m\,,\\ F^{(2)}_{MN} &= 2\partial_{[M}C^{(1)}_{N]} + mB_{MN}\,,\\ F^{(4)}_{MNPQ} &= 4\partial_{[M}C^{(3)}_{NPQ]} - 4H_{[MNP}C^{(1)}_{Q]} + 3mB_{[MN}B_{PQ]}\,.\end{aligned} \tag{C.3}$$

The constant m is the mass parameter of the massive type-IIA theory and the usual massless IIA theory is recovered by setting $m = 0$, in which case the these equations take the form (B.10) above. The string frame metric and spinors are given by,

$$G_{MN} = e^{\phi/2} G'_{MN}\,, \quad \epsilon = e^{\phi/8}\epsilon'\,, \quad \lambda = e^{-\phi/8}\lambda'\,, \quad \Psi_M = e^{\phi/8}(\Psi'_M + \frac{1}{4}\Gamma'_M \lambda')\,.$$

Let us consider the above equations in terms of the positive and negative chirality components of ϵ and other spinors. One can then see that the type-IIA theory described in [20] is the one in which the positive chirality component of ϵ originates in the left-moving worldsheet sector. However, in our conventions for T-duality, we need the IIA in which the positive chirality component of ϵ originates in the right-moving worldsheet sector. This IIA theory is obtained from the one described in [20] by a worldsheet parity transformation that reverses the signs of H_{MNP} and $F^{(2)}$, keeping $F^{(0)}$ and $F^{(4)}$ unchanged. Then the above equations lead to,

$$\delta_\mp \lambda_\pm = \frac{1}{8} e^\phi \left(5F^{(0)} \pm \frac{3}{2!}\Gamma^{M_1M_2} F^{(2)}_{M_1M_2} + \frac{1}{4!}\Gamma^{M_1M_2M_3M_4} F^{(4)}_{M_1M_2M_3M_4} \right) \epsilon_\mp + \cdots\,, \tag{C.4}$$

$$\delta_\mp \Psi_{\pm M} = \frac{1}{8} e^\phi \Big[F^{(0)} \pm \frac{1}{2!}\Gamma^{M_1M_2} F^{(2)}_{M_1M_2} + \frac{1}{4!}\Gamma^{M_1M_2M_3M_4} F^{(4)}_{M_1M_2M_3M_4} \Big] \Gamma_M \epsilon_\mp + \cdots\,. \tag{C.5}$$

The variations $\delta_\pm \lambda_\pm$ and $\delta_\pm \Psi_{\pm M}$ are still given by equations (B.6) and (B.8) though now, ϵ_-, Ψ_{-M} and λ_+ have positive chirality and ϵ_+, Ψ_{+M} and λ_- have negative space-time chirality.

Supersymmetry Variations of NS-NS Fields:

$$\delta_\pm G_{MN} = 2\,\bar\epsilon_\pm\,\Gamma_{(M}\Psi_{\pm N)}\,, \qquad \delta_\pm B_{MN} = \pm 2\,\bar\Psi_{\pm[M}\,\Gamma_{N]}\epsilon_\pm\,, \qquad \delta_\pm \phi = \bar\lambda_\pm \epsilon_\pm\,. \tag{C.6}$$

Here, G_{MN} is the string metric and $()$ denotes symmetrization with unit weight. These equations are valid in both IIA and IIB.

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